

CHEM 5210.001
SPRING 2016

Lecture: WF - 11:00 AM to 12:20 PM - Room ENV 391

Instructor: Martin Schwartz

Office: Chem. Bldg.: Rm. 272

Off. Hrs: Mon through Friday - 10:00 AM to 11:00 + **Anytime**

Office Ph.: 565-3542

Home Ph.: 382-1370

E-mail: marty@unt.edu

Web Site: <http://www.chem.unt.edu/~mschwartz/chem5210/>

or: <http://www.chem.unt.edu/> and navigate (→Faculty→Schwartz→Classes)

I. COURSE MATERIAL

A. Chapter	Title	Approx. Starting Date (Week of)
1.	Introduction and Background To QM	Jan. 20
2.	Quantum Theory	Jan. 27
3.	Particle-in-Box Models	Feb. 3
4.	Rigid-Rotor Models and Angular Momentum Eig.	Feb. 17
5.	Molecular Vibrations and Time Indep. Perturb.	Mar. 2
6.	The Hydrogen Atom	Mar. 9
7.	Multielectron Atoms.....	Mar. 30
8.	Diatomic Molecules	Apr. 13
9.	<i>Ab Initio</i> and Density Functional Methods	Apr. 20
10.	Semiempirical Methods and Applications of Symmetry.....	Apr. 27

B. Text: **No Required Text:** When I've taught CHEM 5210 in the past, students commented that they utilized the PP lectures, homework, old exams to study for tests, and didn't find the textbook particularly useful. Therefore, I do not assign any required text for the course.

I **would** suggest that as we reach specific material (e.g. Particle in Box, Rigid Rotor, Harmonic Oscillator, etc.), you will find it useful to review the material in the quantum mechanics section of any good undergraduate textbook.

II. HOMEWORK

Homework problems will be assigned for each chapter (attached to the Chapter Outline) .

Homework will **not** be collected. However, you are **strongly encouraged** to work the homework, since problems and questions on the exams will be based upon homework and examples worked in class.

Solutions to the homework problems are available on the CHEM 5210 Web Site.

III. EXAMS

A. GENERAL

1. There will be three “hourly” exams. Each hourly exam will count 100 points. You will be given ~1.8 hours (10:00 AM - 11:50 AM) to work each exam. The tests will be in a room (to be determined) in the Chemistry Building.
2. There will be a 2+ hour comprehensive final exam. The Final will count 200 points. The test will be in a room (to be determined) in the Chemistry Building.
3. Either the lowest of the three hourly exams **OR** one-half of the final exam will be **dropped** prior to computing your average.
4. There will **not** be any makeup exams. A missed exam will count as your dropped test (excluding a **well documented** serious illness, requiring hospitalization).
5. If classes are cancelled by the University on the day of a scheduled exam, then the test is automatically scheduled for the next class lecture period.

B. TEST SCHEDULE

Exam #	Date
---------------	-------------

1	Friday, February 19
---	---------------------

2	Friday, March 25
---	------------------

3	Friday, April 22
---	------------------

Final Exam	Monday, May 9, 10:30 AM - 12:30 PM
------------	------------------------------------

We will have the Final Exam in a room (to be determined) in the Chemistry Building. The above time period is the official time. We will arrange extra time for the Final Exam.

IV. COURSE GRADING**A. CALCULATION OF AVERAGE**

Your average will be calculated as a percentage of 400 points. The average will be calculated after dropping the lower of either:

- a) The lowest of the three hourly exams.
- b) One-half of the final exam.

B. GRADING SCALE

A: $\geq 90\%$ B: $\geq 75\%$

V. NOTES

- 1. By University regulations, a grade of "I" cannot be given as a substitute for a failing grade in a course.
- 2. **ADA Compliance:** I am happy to cooperate with the Office of Disability Accommodation to make reasonable accommodations for qualified students with disabilities. If applicable, please present your request, with written verification from the ODA, before the first test.
- 3. Any student caught cheating on an examination will receive an "F" in the course and be reported to the Dean of Students.

CHAPTER 1
INTRODUCTION AND BACKGROUND TO QUANTUM MECHANICS
OUTLINE

Homework Questions Attached

SECT	TOPIC
-------------	--------------

- | | |
|----|---|
| 1. | Problems in Classical Physics |
| 2. | The "Old" Quantum Mechanics (Bohr Theory) |
| 3. | Wave Properties of Particles |
| 4. | Heisenberg Uncertainty Principle |
| 5. | Mathematical Preliminaries |
| 6. | Concepts in Quantum Mechanics |

Chapter 1 Homework

1. Simplify (a) i^2 , (b) i^3 , (c) i^4 , (d) $i \cdot i$, (e) $(1-3i)/(4+2i)$
2. (a) Calculate the energy of one photon of infrared radiation from a Nd:YAG laser, whose wavelength is 1064 nm.
(b) A Nd:YAG laser emits a pulse of 1064 nm radiation of average power 5×10^6 W (J/s) and duration 20 ns. Find the number of photons emitted in this pulse.
3. Find the complex conjugate of (a) -4, (b) $-2i$, (c) $6+3i$, (d) $2e^{-i\pi/5}$.
4. Use the Rydberg expression (appropriately modified for $Z \neq 1$) to calculate the Ionization Energy (in eV) of He^+ . **Note:** $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$

$$\tilde{\nu} = 108,680 \cdot Z^2 \left[\frac{1}{n_L^2} - \frac{1}{n_U^2} \right] \text{ cm}^{-1}$$
5. What is the uncertainty in velocity if we wish to locate an electron within an atom, for which $\Delta x = 50 \text{ pm}$.
6. Calculate the de Broglie wavelength of a helium atom with a kinetic energy of 0.025 eV, in Å (0.025 eV is a typical value of the thermal energy at room temperature).

Some “Concept Question” Topics

Refer to the PowerPoint presentation for explanations on these topics.

- Wavefunction for a free particle

DATA

$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$	$1 \text{ J} = 1 \text{ kg}\cdot\text{m}^2/\text{s}^2$
$\hbar = h/2\pi = 1.05 \times 10^{-34} \text{ J}\cdot\text{s}$	$1 \text{ Å} = 10^{-10} \text{ m}$
$c = 3.00 \times 10^8 \text{ m/fs} = 3.00 \times 10^{10} \text{ cm/s}$	$k \cdot N_A = R$
$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$	$1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg}$
$k = 1.38 \times 10^{-23} \text{ J/K}$	$1 \text{ atm.} = 1.013 \times 10^5 \text{ Pa}$
$R = 8.31 \text{ J/mol}\cdot\text{K}$	$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$
$R = 8.31 \text{ Pa}\cdot\text{m}^3/\text{mol}\cdot\text{K}$	
$m_e = 9.11 \times 10^{-31} \text{ kg}$ (electron mass)	

Chapter 1

Introduction and Background to Quantum Mechanics

Slide 1

The Need for Quantum Mechanics in Chemistry

Without Quantum Mechanics, how would you explain:

- Periodic trends in properties of the elements
- Structure of compounds
e.g. Tetrahedral carbon in ethane, planar ethylene, etc.
- Bond lengths/strengths
- Discrete spectral lines (IR, NMR, Atomic Absorption, etc.)
- Electron Microscopy

Without Quantum Mechanics, chemistry would be a purely empirical science.

PLUS: In recent years, a rapidly increasing percentage of experimental chemists are performing quantum mechanical calculations as an essential complement to interpreting their experimental results.

Slide 2

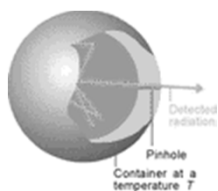
Outline

- Problems in Classical Physics
- The “Old” Quantum Mechanics (Bohr Theory)
- Wave Properties of Particles
- Heisenberg Uncertainty Principle
- Mathematical Preliminaries
- Concepts in Quantum Mechanics

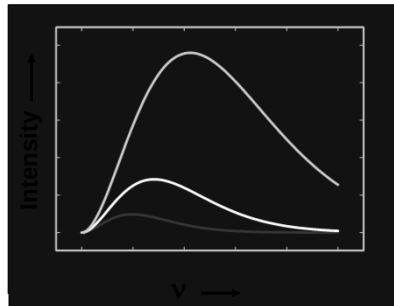
There is nothing new to be discovered in Physics now.
All that remains is more and more precise measurement.
Lord Kelvin (Sir William Thompson), ca 1900

Slide 3

Blackbody Radiation



Heated Metal



Low Temperature: Red Hot

Intermediate Temperature: White Hot

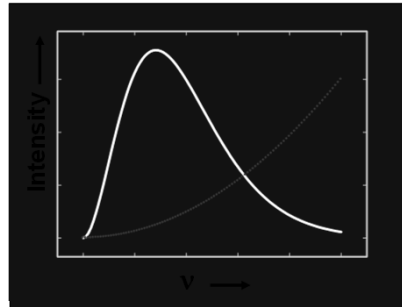
High Temperature: Blue Hot

Slide 4

Rayleigh-Jeans (Classical Physics)

Assumed that electrons in metal oscillate about their equilibrium positions at arbitrary frequency (energy). Emit light at oscillation frequency.

$$\rho(\nu, T) = \frac{8\pi kT}{c^3} \nu^2 = \text{const} \cdot \nu^2$$



The Ultraviolet Catastrophe: $\int_0^{\infty} \rho(\nu, T) d\nu \rightarrow \infty$

Slide 5

Max Planck (1900)

Arbitrarily assumed that the energy levels of the oscillating electrons are quantized, and the energy levels are proportional to ν :

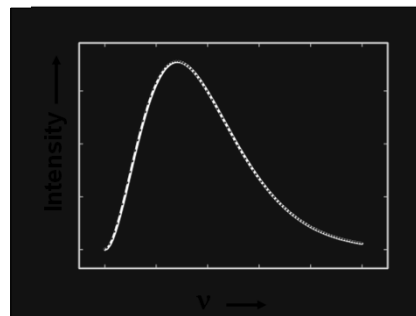
$$\varepsilon = h(n\nu)$$

$$n = 1, 2, 3, \dots$$

h = empirical constant

He derived the expression:

$$\rho(\nu, T) = \frac{8\pi h}{c^3} \nu^3 \frac{1}{e^{h\nu/kT} - 1}$$

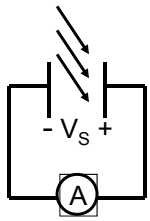


Expression matches experimental data perfectly for

$$h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s} \quad [\text{Planck's Constant}]$$

Slide 6

The Photoelectric Effect

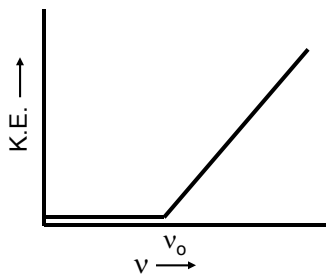


Kinetic Energy of ejected electrons can be measured by determining the magnitude of the “stopping potential” (V_s) required to stop current.

Observations

Low frequency (red) light: $\nu < \nu_0$ - No ejected electrons (no current)

High frequency (blue) light: $\nu > \nu_0$ - K.E. of ejected electrons $\propto \nu$



Slide 7

Photons

Einstein (1903) proposed that light energy is quantized into “packets” called photons.

$$E_{ph} = h\nu$$

Explanation of Photoelectric Effect

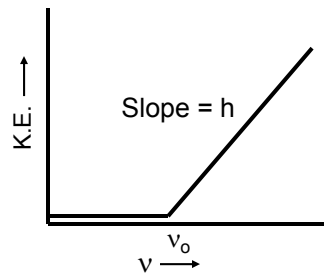
$$E_{ph} = h\nu = \phi + \text{K.E.}$$

ϕ is the metal's “work function”: the energy required to eject an electron from the surface

$$\text{K.E.} = h\nu - \phi = h\nu - h\nu_0$$

$$\nu_0 = \phi / h$$

Predicts that the slope of the graph of K.E. vs. ν is h (Planck's Constant) in agreement with experiment !!



Slide 8

Equations Relating Properties of Light

**Wavelength/
Frequency:** $\lambda \cdot \nu = c$

Wavenumber: $\tilde{\nu} = \frac{1}{\lambda (cm)}$ Units: cm^{-1}

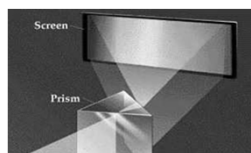
$\nu = \frac{c}{\lambda} = c \cdot \frac{1}{\lambda} = c\tilde{\nu}$ c must be in cm/s

Energy: $E_{ph} = h\nu = \frac{hc}{\lambda} = hc\tilde{\nu}$

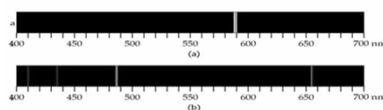
You should know these relations between the properties of light. They will come up often throughout the course.

Slide 9

Atomic Emission Spectra



← Sample
Heat



When a sample of atoms is heated up, the excited electrons emit radiation as they return to the ground state.

The emissions are at discrete frequencies, rather than a continuum of frequencies, as predicted by the Rutherford planetary model of the atom.

Slide 10

Hydrogen Atom Emission Lines

UV Region:
(Lyman Series) $\tilde{\nu} = \frac{1}{\lambda} = 108,680 \left[1 - \frac{1}{n^2} \right] \text{ cm}^{-1}$ $n = 2, 3, 4 \dots$

Visible Region:
(Balmer Series) $\tilde{\nu} = \frac{1}{\lambda} = 108,680 \left[0.25 - \frac{1}{n^2} \right] \text{ cm}^{-1}$ $n = 3, 4, 5, \dots$

Infrared Region:
(Paschen Series) $\tilde{\nu} = \frac{1}{\lambda} = 108,680 \left[0.111 - \frac{1}{n^2} \right] \text{ cm}^{-1}$ $n = 4, 5, 6 \dots$

General Form (Johannes Rydberg)

$$\tilde{\nu} = \frac{1}{\lambda} = R_H \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \text{ cm}^{-1} \quad \begin{array}{l} n_1 = 1, 2, 3 \dots \\ n_2 > n_1 \end{array}$$

$$R_H = 108,680 \text{ cm}^{-1}$$

Slide 11

Outline

- Problems in Classical Physics
- **The “Old” Quantum Mechanics (Bohr Theory)**
- Wave Properties of Particles
- Heisenberg Uncertainty Principle
- Mathematical Preliminaries
- Concepts in Quantum Mechanics

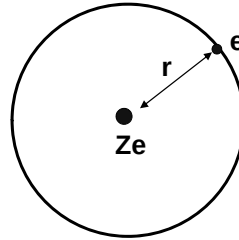
Slide 12

The “Old” Quantum Theory

Niels Bohr (1913)

Assumed that electron in hydrogen-like atom moved in circular orbit, with the centripetal force (mv^2/r) equal to the Coulombic attraction between the electron (with charge e) and nucleus (with charge Ze).

$$f = ma = m \frac{v^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{Ze \cdot e}{r^2}$$



He then arbitrarily assumed that the “angular momentum” is quantized.

$$\vec{L} = \vec{r} \times \vec{p} = mvr = n\hbar \quad n = 1, 2, 3, \dots$$

$$\hbar = \frac{h}{2\pi} \quad (\text{Dirac's Constant})$$

Why??

Because it worked.

Slide 13

$$m \frac{v^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{Ze \cdot e}{r^2}$$

$$mvr = n\hbar$$

It can be shown

$$r = \frac{4\pi\epsilon_0 \hbar^2 n^2}{me^2 Z} = n^2 \frac{a_0}{Z}$$

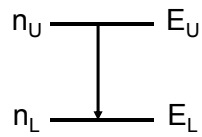
$$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{me^2} = 0.529 \text{ \AA} \quad (\text{Bohr Radius})$$

$$E = K.E. + P.E. = \frac{1}{2}mv^2 - \frac{Ze^2}{4\pi\epsilon_0} \frac{1}{r}$$

$$E = -\frac{me^4 Z^2}{8\epsilon_0^2 h^2} \frac{1}{n^2} = -\frac{\kappa}{n^2} = -2.181 \times 10^{-18} \frac{1}{n^2} \text{ Joules}$$

$$n = 1, 2, 3, \dots$$

Slide 14

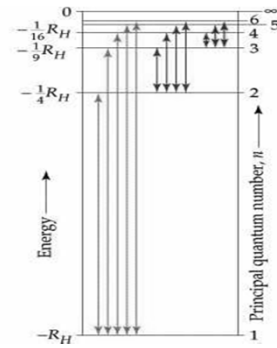


$$E_{ph} = \Delta E = E_U - E_L$$

$$\Delta E = \left[-\frac{\kappa}{n_U^2} \right] - \left[-\frac{\kappa}{n_L^2} \right]$$

$$\Delta E = \kappa \left[\frac{1}{n_L^2} - \frac{1}{n_U^2} \right]$$

$$= 2.181 \times 10^{-18} \left[\frac{1}{n_L^2} - \frac{1}{n_U^2} \right] \text{ Joules}$$

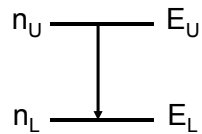


Lyman Series: $n_L = 1$

Balmer Series: $n_L = 2$

Paschen Series: $n_L = 3$

Slide 15



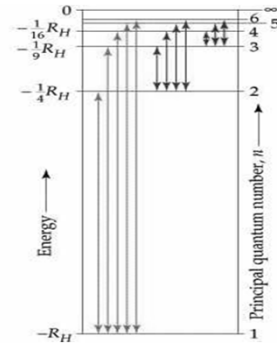
$$\Delta E = \kappa \left[\frac{1}{n_L^2} - \frac{1}{n_U^2} \right]$$

$$= 2.181 \times 10^{-18} \left[\frac{1}{n_L^2} - \frac{1}{n_U^2} \right] \text{ Joules}$$

$$\tilde{\nu} = \frac{E_{ph}}{hc} = \frac{\Delta E}{hc} = 109,800 \left[\frac{1}{n_L^2} - \frac{1}{n_U^2} \right] \text{ cm}^{-1}$$

Close to $R_H = 108,680 \text{ cm}^{-1}$

Get perfect agreement if replace electron mass (m) by reduced mass (μ) of proton-electron pair.



Slide 16

The Bohr Theory of the atom ("Old" Quantum Mechanics) works perfectly for H (as well as He^+ , Li^{2+} , etc.).

And it's so much EASIER than the Schrödinger Equation.

The only problem with the Bohr Theory is that it fails as soon as you try to use it on an atom as "complex" as helium.

Slide 17

Outline

- Problems in Classical Physics
- The "Old" Quantum Mechanics (Bohr Theory)
- **Wave Properties of Particles**
 - Heisenberg Uncertainty Principle
 - Mathematical Preliminaries
- Concepts in Quantum Mechanics

Slide 18

Wave Properties of Particles

The de Broglie Wavelength

Louis de Broglie (1923): If waves have particle-like properties (photons, then particles should have wave-like properties.

Photon wavelength-momentum relation

$$E = h\nu = \frac{hc}{\lambda} \quad \text{and} \quad E = mc^2$$

$$\lambda = \frac{hc}{E} = \frac{hc}{mc^2} = \frac{h}{mc} = \frac{h}{p}$$

de Broglie wavelength of a particle

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Slide 19

What is the de Broglie wavelength of a 1 gram marble traveling at 10 cm/s

$$h = 6.63 \times 10^{-34} \text{ J-s}$$

$$\lambda = 6.6 \times 10^{-30} \text{ m} = 6.6 \times 10^{-20} \text{ \AA} \text{ (insignificant)}$$

What is the de Broglie wavelength of an electron traveling at 0.1 c (c=speed of light)?

$$c = 3.00 \times 10^8 \text{ m/s}$$

$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$\lambda = 2.4 \times 10^{-11} \text{ m} = 0.24 \text{ \AA}$$

(on the order of atomic dimensions)

Slide 20

Reinterpretation of Bohr's Quantization of Angular Momentum

$$\vec{L} = \vec{r} \times \vec{p} = mvr = n\hbar \quad n = 1, 2, 3, \dots \quad \hbar = \frac{h}{2\pi} \quad (\text{Dirac's Constant})$$

$$mvr = \frac{nh}{2\pi}$$



$$2\pi r = n \frac{h}{mv} = n \frac{h}{p}$$



$$2\pi r = n\lambda$$

The circumference of a Bohr orbit must be a whole number of de Broglie “standing waves”.

Slide 21

Outline

- Problems in Classical Physics
- The “Old” Quantum Mechanics (Bohr Theory)
- Wave Properties of Particles
- **Heisenberg Uncertainty Principle**
- Mathematical Preliminaries
- Concepts in Quantum Mechanics

Slide 22

Heisenberg Uncertainty Principle

Werner Heisenberg: 1925

It is not possible to determine both the position (x) and momentum (p) of a particle precisely at the same time.

$$\Delta p \cdot \Delta x \geq \frac{\hbar}{2}$$

Δp = Uncertainty in momentum

Δx = Uncertainty in position

There are a number of pseudo-derivations of this principle in various texts, based upon the wave property of a particle. We will not give one of these derivations, but will provide examples of the uncertainty principle at various times in the course.

Slide 23

Calculate the uncertainty in the position of a 5 Oz (0.14 kg) baseball traveling at 90 mi/hr (40 m/s), assuming that the velocity can be measured to a precision of 10^{-6} percent.

$$\Delta x = 9.4 \times 10^{-28} \text{ m}$$

$$h = 6.63 \times 10^{-34} \text{ J-s}$$

$$\hbar = 1.05 \times 10^{-34} \text{ J-s}$$

Calculate the uncertainty in the momentum (and velocity) of an electron ($m_e = 9.11 \times 10^{-31} \text{ kg}$) in an atom with an uncertainty in position, $\Delta x = 0.5 \text{ \AA} = 5 \times 10^{-11} \text{ m}$.

$$\Delta p = 1.05 \times 10^{-24} \text{ kgm/s}$$

$$\Delta v = 1.15 \times 10^6 \text{ m/s } (= 2.6 \times 10^6 \text{ mi/hr})$$

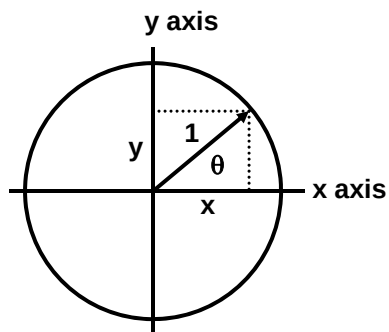
Slide 24

Outline

- Problems in Classical Physics
- The “Old” Quantum Mechanics (Bohr Theory)
- Wave Properties of Particles
- Heisenberg Uncertainty Principle
- **Mathematical Preliminaries**
- Concepts in Quantum Mechanics

Slide 25

Math Preliminary: Trigonometry and the Unit Circle



$$\cos(\theta) = x$$

$$\sin(\theta) = y$$

From the unit circle,
it's easy to see that: $\cos(-\theta) = \cos(\theta)$
 $\sin(-\theta) = -\sin(\theta)$

$$\sin(0^\circ) = 0$$

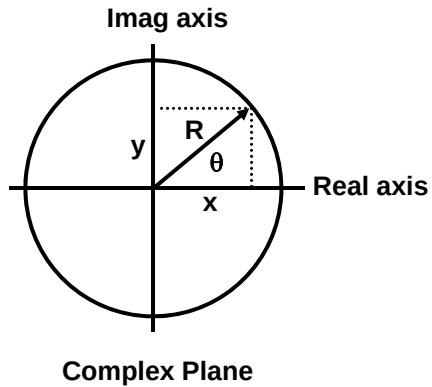
$$\cos(180^\circ) = -1$$

$$\sin(90^\circ) = 1$$

$$\cos(270^\circ) = 0$$

Slide 26

Math Preliminary: Complex Numbers



$$i = \sqrt{-1}$$

Euler Relations

$$e^{i\theta} = \cos(\theta) + i \sin(\theta)$$

$$e^{-i\theta} = \cos(\theta) - i \sin(\theta)$$

Complex number (z)

$$z = x + iy \quad \text{or} \quad z = Re^{i\theta}$$

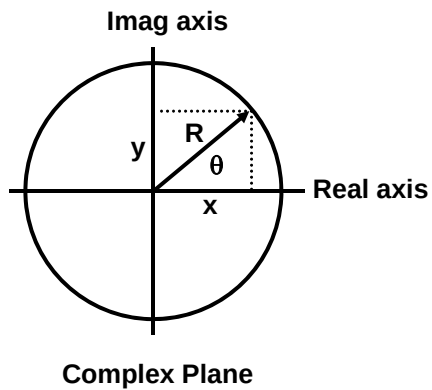
where $x = R \cos(\theta)$ and $y = R \sin(\theta)$

Complex conjugate (z^*)

$$z^* = x - iy \quad \text{or} \quad z^* = Re^{-i\theta}$$

Slide 27

Math Preliminary: Complex Numbers



$$z = x + iy \quad \text{or} \quad z = Re^{i\theta}$$

where $x = R \cos(\theta)$ and $y = R \sin(\theta)$

Magnitude of a Complex Number

$$|z| = \sqrt{|z|^2}$$

$$|z|^2 = z^* z = (x + iy)(x - iy) = x^2 + y^2$$

or

$$|z|^2 = z^* z = (Re^{i\theta})(Re^{-i\theta}) = R^2$$

Slide 28

Outline

- Problems in Classical Physics
- The “Old” Quantum Mechanics (Bohr Theory)
- Wave Properties of Particles
- Heisenberg Uncertainty Principle
- Mathematical Preliminaries
- **Concepts in Quantum Mechanics**

Slide 29

Concepts in Quantum Mechanics

Erwin Schrödinger (1926): If, as proposed by de Broglie, particles display wave-like properties, then they should satisfy a wave equation similar to classical waves. He proposed the following equation.

One-Dimensional Time Dependent Schrödinger Equation

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V(x,t)\Psi$$

$$\hbar = \frac{h}{2\pi} \text{ (Dirac's Constant)}$$

Ψ is the wavefunction

m = mass of particle

$|\Psi|^2 = \Psi^* \Psi$ is the probability of finding the particle between

$V(x,t)$ is the potential energy

x and $x + dx$

Slide 30

Wavefunction for a free particle

$$-\infty \longleftarrow \text{---} \bullet \text{---} \longrightarrow +\infty$$

$$V(x,t) = \text{const} = 0$$

~~$$\Psi = C \cos(kx - \omega t)$$~~ where $k = \frac{2\pi}{\lambda}$ and $E = h\nu = \hbar\omega$

Classical Traveling Wave

$$\text{For a particle: } \lambda = \frac{h}{p}$$

Unsatisfactory because $|\Psi|^2 = C^2 \cos^2(kx - \omega t) \neq \text{constant}$

The probability of finding the particle at any position
(i.e. any value of x) should be the same

$$\Psi = Ce^{i(kx - \omega t)} \text{ is satisfactory}$$

$$\text{Note that: } |\Psi|^2 = \Psi^* \Psi = C^2 = \text{constant}$$

Slide 31

"Derivation" of Schrödinger Eqn. for Free Particle

$$\Psi = Ce^{i(kx - \omega t)} \quad \text{where} \quad k = \frac{2\pi}{\lambda} \quad \text{and} \quad \lambda = \frac{h}{p}$$

$$E = h\nu = \hbar\omega$$

on board

$$\Psi = Ce^{\frac{i}{\hbar}(px - Et)}$$

on board

$$E\Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

$$\frac{p^2}{2m} \Psi = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2}$$

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2}$$

Schrödinger Eqn.
for $V(x,t) = 0$

Slide 32

Note: We cannot actually derive Quantum Mechanics or the Schrödinger Equation.

In the last slide, we gave a rationalization of how, if a particle behaves like a wave and λ is given by the de Broglie relation, then the wavefunction, Ψ , satisfies the wave equation proposed by Erwin Schrödinger.

Quantum Mechanics is not “provable”, but is built upon a series of postulates, which will be discussed in the next chapter.

The validity of the postulates is based upon the fact that Quantum Mechanics WORKS. It correctly predicts the properties of electrons, atoms and other microscopic particles.

Slide 33